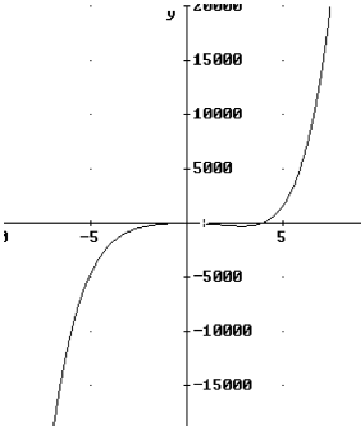
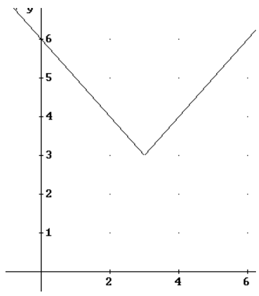
1. **Do not use the procedure for finding absolute extrema blindly**.

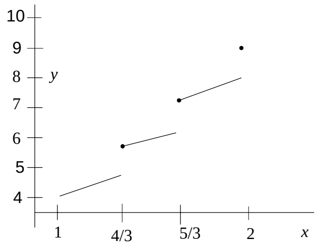


a. $g'(x) = 5x^4 - 120x = 5x(x - 2\cdot 3^{\frac{1}{3}})(x^2 + 2\cdot 3^{\frac{1}{3}}x + 4\cdot 3^{\frac{2}{3}})$. Therefore, there are only 2 critical points of g in $(-6, 8)$, which is 0 and $2\cdot 3^{\frac{1}{3}}$. Now $g(0) = 3$, $g(-6) = -9933$ and $g(2\cdot 3^{\frac{1}{3}}) = 3 - 144\cdot 3^{\frac{2}{3}} \doteqdot -296.532$ and $g(8) = 28931$. Thus the absolute maximum is 28931 and the absolute minimum is -9933.

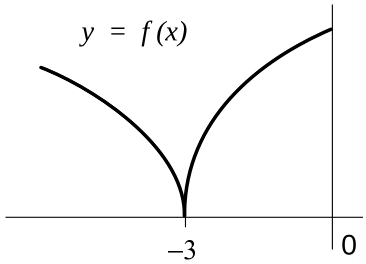


b. $h(x) = |x-3| + 3 = \begin{cases} x, & 3 \le x < 5 \\ 6 - x, & 0 < x < 3 \end{cases}$. Note that h is continuous on (0, 5) and $h'(x) = \begin{cases} -1, & 0 < x < 3 \\ 1, & 3 < x < 5 \end{cases}$ Therefore, h is decreasing on (0, 3] and increasing on [3, 5). Thus, $h(3) = 3$ is the absolute minimum. Since 0 and 5 are not in (0,5), the absolute maximum of h on (0,5) does not exist.

C. **Simplify the given function first.**



If x is in (1, 2], then $k(x) = \begin{cases} 2x+2, & 1 < x < \frac{4}{3} \\ 2x+3, & \frac{4}{3} \le x < \frac{5}{3} \\ 2x+4, & \frac{5}{3} \le x < 2 \\ 2x+5 = 9, & x = 2 \end{cases}$. Thus the absolute maximum of k on (1, 2] is 9 and the absolute minimum does not exist.



d. $f(x)$ is continuous on $[-5,0]$ and for $x \ne -3$, $f'(x) = \frac{2}{3}(3+x)^{-\frac{1}{3}}$. Indeed the derivative of f at -3 does not exist. -3 is the only crtitical point since $f'(x) \ne 0$ for $x \ne -2$. Now $f(-5) = 2^{\frac{2}{3}}$, $f(-3) = 0$, $f(0) = 3^{\frac{2}{3}}$. Thus, the absolute minimum is 0 and the absolute maximum is $3^{\frac{2}{3}}$.

2. a. $f(x)$ is continuous on [0, 9/4]. $f'(x) = \dfrac{2(x-2)(x-3)}{(2x-5)^2}$.

Thus f is differentiable on (0, 9/4) and the only critical point in the interval is 2.
Now $f(2) = 2$, $f(0) = 6/5$, $f(9/4) = 15/8$.
(Note that f is not defined at $x = 5/2$ as it is not in the interval [0,9/4].)
Therefore, the absolute maximum is 2 and the absolute minimum is 6/5.

b. $g'(x) = \begin{cases} 3x^2 - 3, & 0 < x < 2 \\ 2x - 5, & 2 < x < 3 \end{cases}$.

Thus when $x = 1$ or $x = \frac{5}{2}$, $g'(x) = 0$. $g(1) = 1 - 3 + 5 = 3$.

$$\lim_{h\to 0^-} \tfrac{g(2+h)-g(2)}{h} = \lim_{h\to 0^-} \tfrac{(2+h)^3-3(2+h)+5-7}{h} = \lim_{h\to 0^-} \tfrac{(9h+6h^2+h^3)}{h} = 9.$$

$$\lim_{h\to 0^+} \tfrac{g(2+h)-g(2)}{h} = \lim_{h\to 0^+} \tfrac{(2+h)^2-5(2+h)+13-7}{h} = \lim_{h\to 0^+} \tfrac{h^2-h}{h} = -1.$$

Therefore, the left and the right derivatives are not the same. So g is not differentiable at 2. Hence the critical points of g on [0, 3] are 5/2, 1 and 2.

Now $g(0) = 5$, $g(\frac{5}{2}) = (\frac{5}{2})^2 - 5\cdot(\frac{5}{2}) + 13 = 6\frac{3}{4}$, $g(2) = 7$, g(1) = 3 and $g(3) = 3^2 - 5\cdot 3 + 13 = 7$. Thus the absolute maximum is 7 and the absolute minimum is 3.

3. a. f is continuous on [0, 32]. Since f is differentiable on $(0,\infty)$, f is differentiable on (0, 32). Now $f(0) = f(32) = 0$. Thus the conditions for *Rolle's Theorem* are satisfied. Therefore, by *Rolle's Theorem,* there exists a point c in (0, 32) such that $f'(c) = 0$.

Indeed $f'(x) = \frac{3}{5}x^{-\frac{2}{5}} - 2\cdot\frac{2}{5}x^{-\frac{3}{5}} = \frac{1}{5}x^{-\frac{3}{5}}(3x^{\frac{1}{5}} - 4) = 0$ if $x = (\frac{4}{3})^5$.

b. Note that g is continuous at 1 and so continuous on $[-\sqrt{3}, 7/4]$. Note also that $g(-\sqrt{3}) = -1 \neq g(7/4) = 0$. Also g is not differentiable at $x = 1$ because

$$\lim_{h\to 0^+} \tfrac{g(1+h)-g(1)}{h} = \lim_{h\to 0^+} \tfrac{4(1+h)-7-(-3)}{h} = 4 \neq \lim_{h\to 0^-} \tfrac{g(1+h)-g(1)}{h} = \lim_{h\to 0^-} \tfrac{(1+h)^2-4-(-3)}{h} = 2.$$

Thus g does not satisfy the conditions of *Rolle's Theorem* for the interval $[-\sqrt{3}, 7/4]$.

But the conditions of *Rolle's Theorem* are satisfied for g on the interval [–1, 1] because

1. $g(-1) = g(1) = -3$,

2. g is continuous on [–1, 1] and

3. g is differentiable on (–1, 1).

Therefore, by *Rolle's Theorem*, there is a point c in (–1, 1) such that $g'(c) = 0$.

Indeed $g'(x) = \begin{cases} 2x, & x < 1 \\ 4, & x > 1 \end{cases}$ and $g'(0) = 0$.

4. Let $f(x) = \sqrt[3]{x}$ for $x \in [27, 28]$. Then $f'(x) = \dfrac{1}{3}(x)^{-2/3}$. Thus by the *Mean Value Theorem*, there is a point c in the open interval (27,28) such that

$$\frac{\sqrt[3]{28} - \sqrt[3]{27}}{28 - 27} = f'(c) = \frac{1}{3}(c)^{-2/3}.$$

Therefore, using the fact that $27 < c < 28$, we have $27^{2/3} < c^{2/3} < 28^{2/3}$.

Thus, $\frac{1}{3}28^{-2/3} < \frac{1}{3}c^{-2/3} = f'(c) < \frac{1}{3}27^{-2/3}$., i.e.

$$\frac{1}{3\cdot(\sqrt[3]{28})^2} < \sqrt[3]{28} - 3 < \frac{1}{3\cdot(\sqrt[3]{27})^2} = \frac{1}{27}.$$

But , $$\frac{1}{3\cdot(\sqrt[3]{28})^2} > \frac{1}{\sqrt[3]{28}\cdot(\sqrt[3]{28})^2} = \frac{1}{28}$$

Thus, $\frac{1}{28} < \sqrt[3]{28} - 3 < \frac{1}{27}$.

5. Let $g(x) = \frac{x^6}{6} - \frac{x^5}{5} + \frac{x^4}{2} - \frac{2x^3}{3} + \frac{x^2}{2} - x - 4 = 0$. Then $g'(x) = x^5 - x^4 + 2x^3 - 2x^2 + x - 1$.
   Then $g'(x) = (x-1)(x^2+1)^2$.
   (a) If g has two zeros, say a and b (assuming $a < b$), in the interval (3/2,2), then by *Rolle's Theorem* there would be a point $c \in (a, b) \subseteq (3/2, 2)$ with $g'(c) = 0$ which contradicts $g'(x) > 0$ for all $x > 1$.
   (b) Now g is continuous on [3/2, 2], as it is a polynomial function, $g(3/2) = -\frac{2377}{640} < 0$, $g(2) = \frac{44}{15} > 0$ so that by the *Intermediate Value Theorem*, there exists a c in (3/2, 2) such that $g(c) = 0$.
   (c) From parts (*a*) and (*b*), $g(x) = 0$ has exactly one root in (3/2, 2).

6. Suppose f is a differentiable function on **R** and $f(0) = -1$, $f(2) = 4$. Then f is continuous on **R**. Now, by the Mean Value Theorem applied to f on [0, 2] , there is a real number $c \in (0, 2)$ such that $\frac{f(2)-f(0)}{2-0} = f'(c)$, i.e. $f'(c) = 5/2$, contradicting the given hypothesis that $f'(x) \le 2$ for all real numbers x.
   Therefore, there is no such function.

7. Since $f'(x) = 4$ for all x in **R**, f is continuous on **R**.
   We consider two cases (i) $x > 1$ and (ii) $x < 1$.
   Case (i) $x > 1$. Consider f on $[1,x]$, then f is continuous on $[1,x]$ and differentiable on $(1,x)$.
   Thus by the Mean Value Theorem, we have $\frac{f(x)-f(1)}{x-1} = f'(c)$, for some c in $(1,x)$;
   I.e. $\frac{f(x)-f(1)}{x-1} = 4$. This gives $f(x) - 3 = 4x - 4$ and so $f(x) = 4x - 1$
   Case (ii) $x < 1$. Consider f on $[x,1]$, then f is continuous on $[x,1]$ and differentiable on $(x,1)$.
   Thus by the Mean Value Theorem, we have $\frac{f(1)-f(x)}{1-x} = f'(c)$, for some c in $(x,1)$;
   I.e. $\frac{f(1)-f(x)}{1-x} = 4$. This gives gain $f(x) = 4x - 1$
   Note that when $x = 1$, $4x - 1 = 3$.
   Therefore, $f(x) = 4x - 1$ for all real numbers x.