National University of Singapore

Department of Mathematics

Level 1000 (2005/2006) Semester 2 MA1102R Calculus Tutorial Set 8

1. Find an antiderivative.of $f(x) = \begin{cases} 4x^3 + x^2 + 1, x > 2 \\ \frac{2x}{\sqrt{12+x^2}} + 5x + 26, x \le 2 \end{cases}$ i.e. find $\int f(x)dx$.

2. Given that $f'(x) = \frac{\sin(5x)}{\sqrt{5 - 2\cos(5x)}}$ and $f(0) = 1$, find $f(x)$.

3. Use Riemann sums to evaluate $\int_1^2 \frac{1}{x^2}dx$. (Hint: $\frac{1}{(1+\frac{i}{n})(1+\frac{i+1}{n})} \le \frac{1}{(1+\frac{i}{n})^2} \le \frac{1}{(1+\frac{i-1}{n})(1+\frac{i}{n})}$.)

4. Evaluate the following as definite integrals.

   a. $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{n+5i}{n^3}}$. b. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \sin\left(\frac{\pi i}{7n}\right)$.

5. Evaluate $\int_0^2 [t^3 + 2]dt$., where $[x]$ denotes the greatest integer $\le x$.

6 Let the function f be continuous on $[0, \infty)$. Show that $\int_0^{\pi} x f(\sin(x))dx = \frac{\pi}{2} \int_0^{\pi} f(\sin(x))dx$.

   Hence, or otherwise, compute $\int_0^{\pi} x \sin^4(x)dx$.

   (Hint: Use the substitution $x = \pi - t$ and the identity $\sin(\pi - t) = \sin(t)$. You may use the identities: $\cos^2(x) = \frac{1}{2}(1 + \cos(2x)); \sin^2(x) = \frac{1}{2}(1 - \cos(2x))$.)

7. Find the following derivatives.

   a. $\frac{d}{dx}\int_1^x \frac{1}{1+t^3+t^6}dt$. b. $\frac{d}{dx}\int_1^{\sin(2x)} \frac{1}{1+t^5+t^{10}}dt$. c. $\frac{d}{dx}\int_{-3x}^{\sin(x^2)} \frac{1}{1+t^3+t^6}dt$.

8. a. Find the area of the region bounded by the curves $y = x^2$ and $x^2 = 20 - y$. Draw a sketch of the region.

   b. Sketch the **region between** the curves $y = \cos(x)$ and $y = \sin(x)$ and bounded by the lines $x = 0$ and $x = 2\pi$. Find the area of this region.

9. Use the *Mean Value Theorem* for integrals to prove that

$$\frac{2}{3251} \le \int_3^5 \frac{1}{x^5 + x^3 + 1}dx \le \frac{2}{271}$$

10. Find $(f^{-1})'(d)$ for the given f and d.

    a. $f(x) = \sqrt{10x+1}$, $x \ge -\frac{1}{10}$; $d = 1$. b. $f(x) = \frac{1}{2}\sec(x)$, $0 \le x < \frac{\pi}{2}$; $d = \frac{1}{\sqrt{3}}$.

    c. $f(x) = x^2 - 5x - 10$, $x \ge \frac{5}{2}$; $d = 4$.